The world's most consistent and transparent Life Cycle Inventory database

Uncertainty correlation and Monte Carlo sampling in LCA

LCAXV, Vancouver, Canada

07 October 2015

Guillaume Bourgault, Ph.D

Project Manager

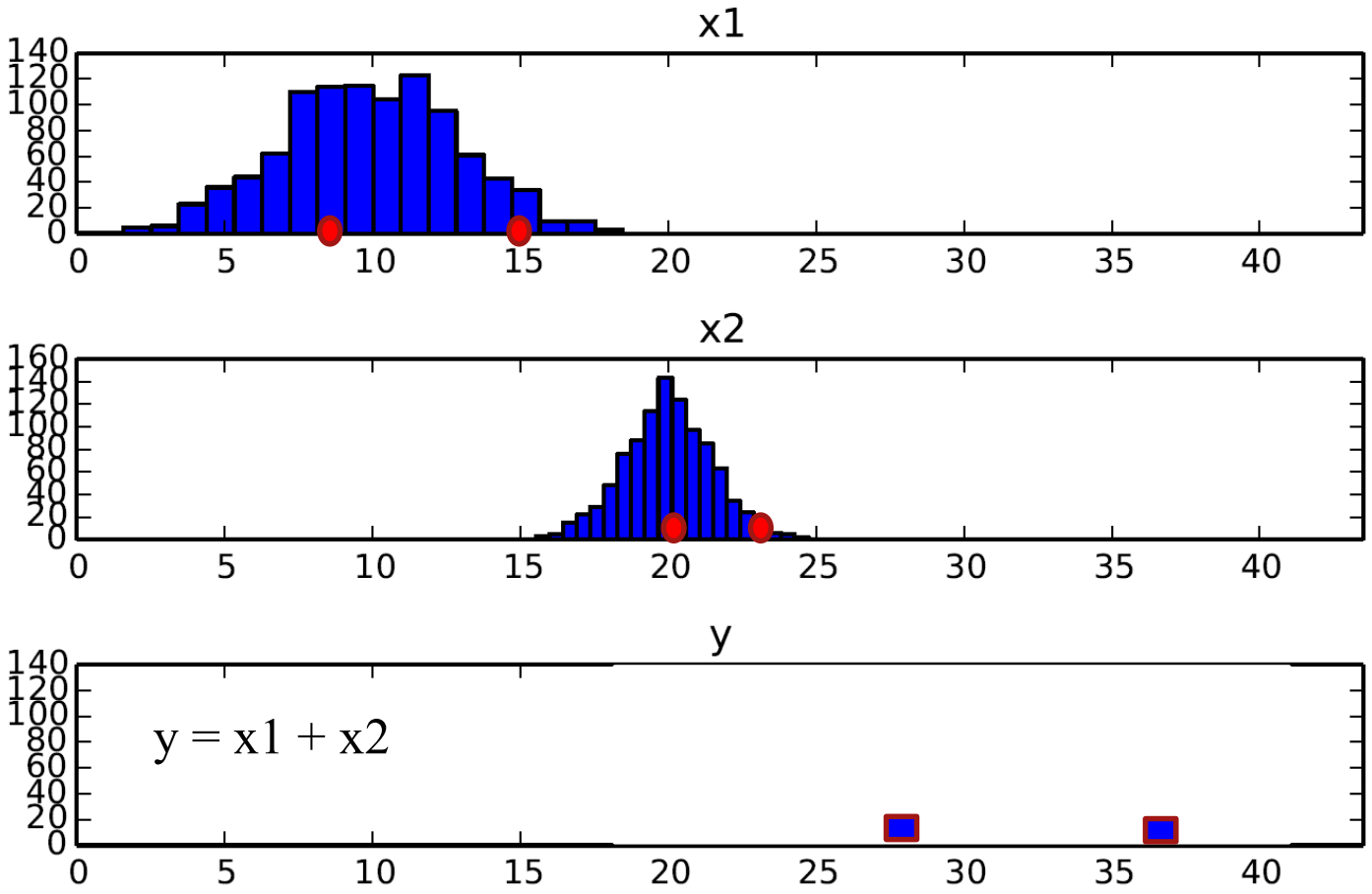
ecoinvent

Agroscope EMPA EPFL ETH PSI

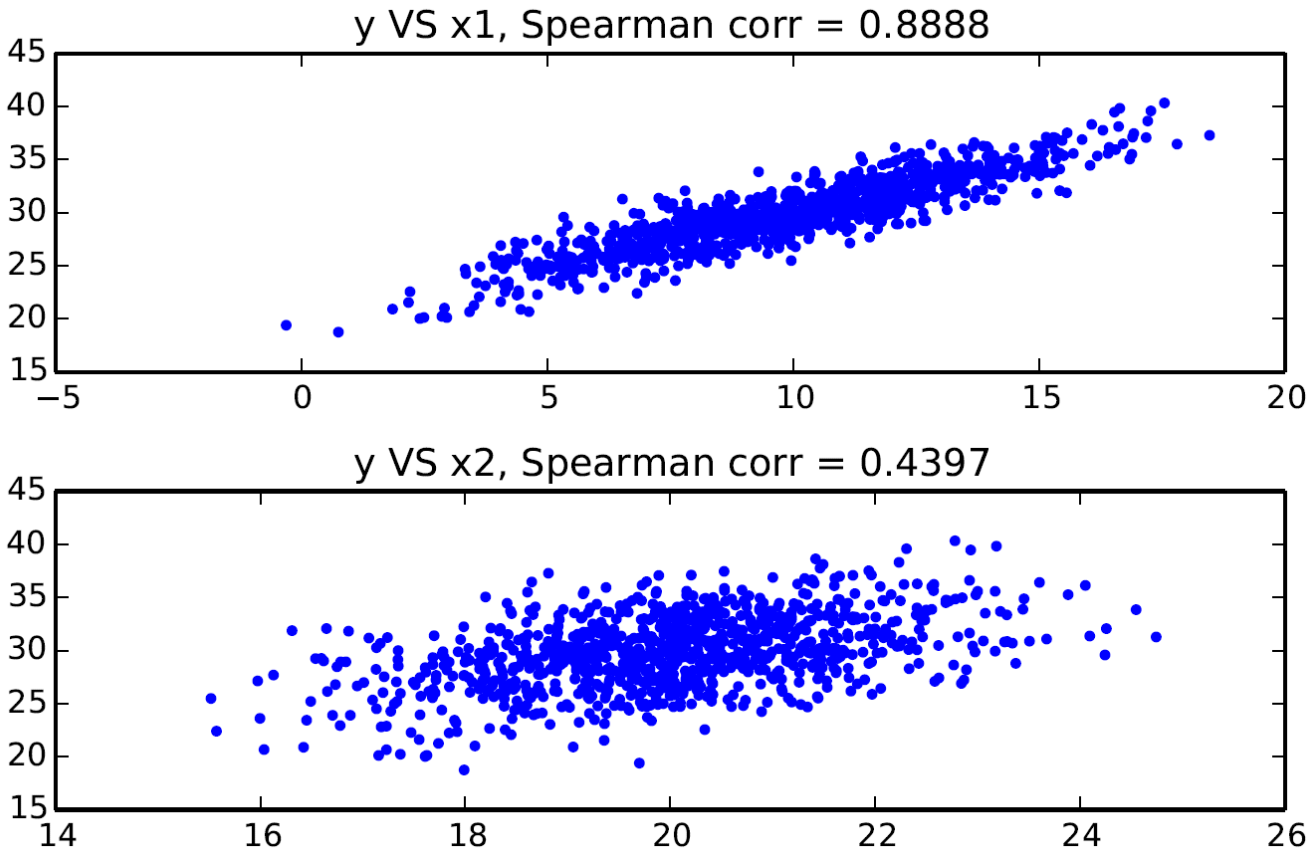
Outline

- What is Monte Carlo and why we use it?
- What correlation are we tackling today?
- How beta and Dirichlet distributions can help?
- Ignoring correlation overestimates uncertainty

What is Monte Carlo?



What is Monte Carlo?



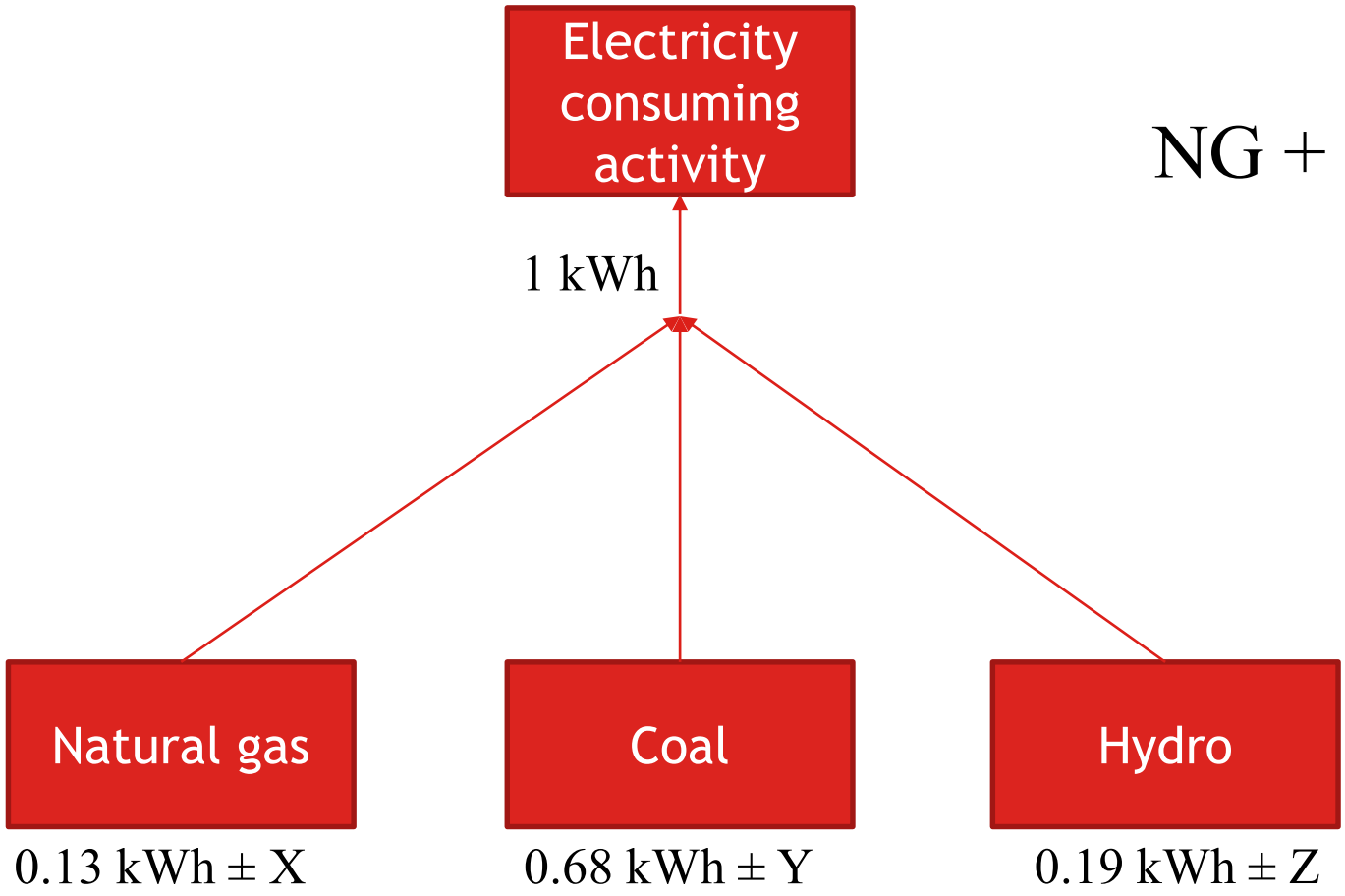
Why Monte Carlo?

- What is the uncertainty of the model output?
  - “All models are wrong, but some are useful”. How useful is your model?
  - Decision makers appreciate the information
- What parameter of the model drives the output uncertainty?
  - Learn something about the model
  - Recommendations on model use
  - Guide data collection
  - Restructure the model: more parameters necessary?

Why Monte Carlo?

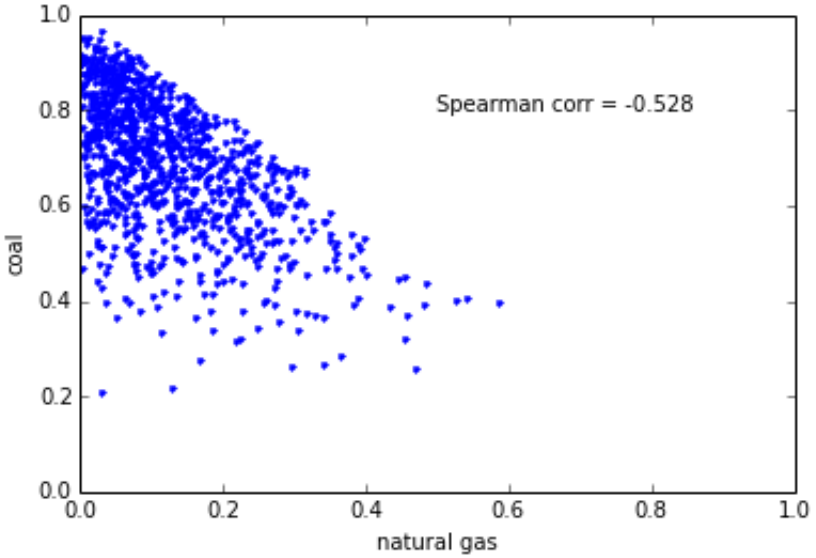
- Simpler models: analytical tools
  - Cumbersome to impossible for large models
  - Taylor series expansion: ill-adapted to large uncertainties in LCIA
- Advantage of Monte Carlo
  - Variables are sampled on their entire domain
  - All variables change at once: captures interactions between parameters
- Drawback of Monte Carlo
  - Computationally expensive (time and memory)
  - Requires coding skills

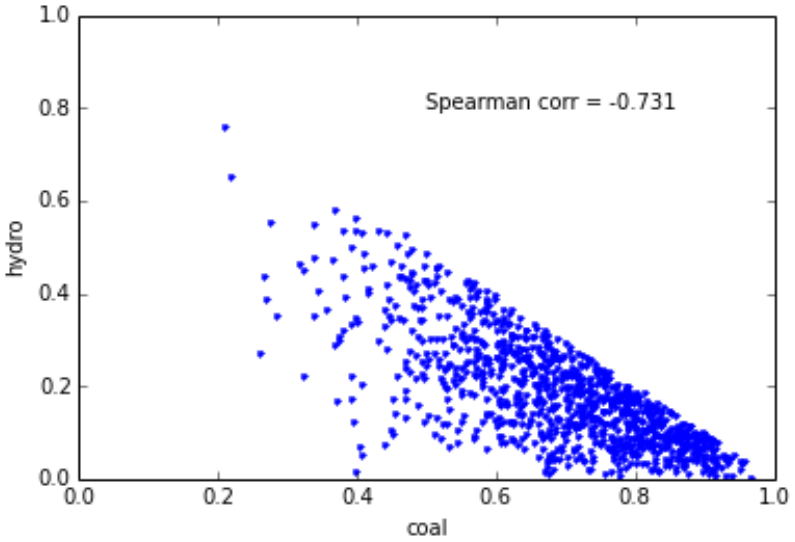
Correlation in LCI

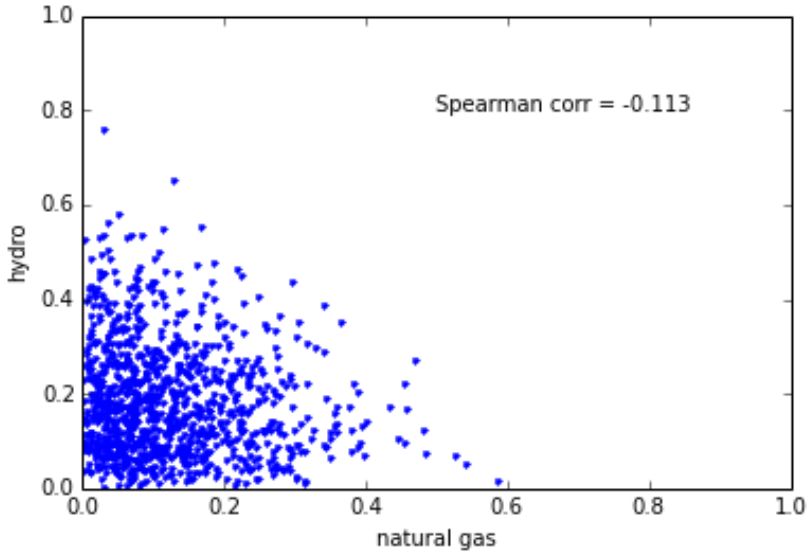


$$NG + C + H = 1$$

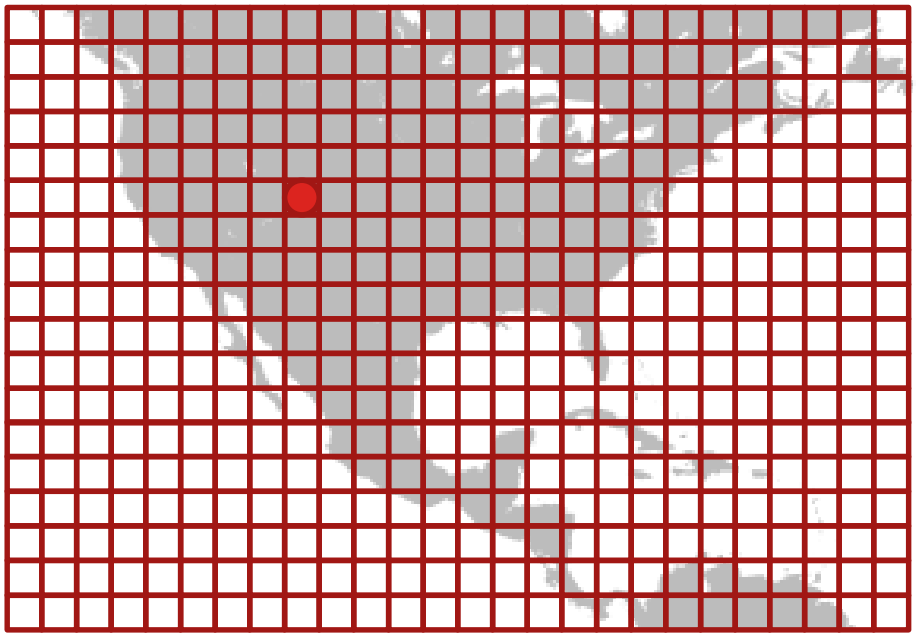
Correlation in LCI







Correlation in LCIA



Fate factor
FF_{ij} = mass transported
from source cell (i)
to all other cells (j).

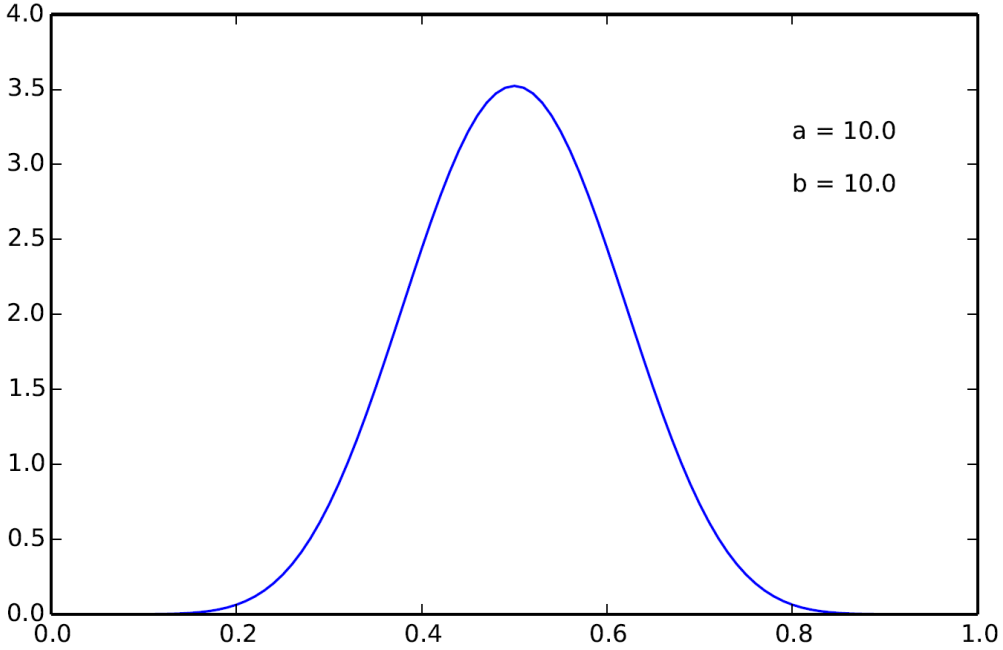
$$1 \geq FF_{ij} \; \forall \; j \geq 0$$

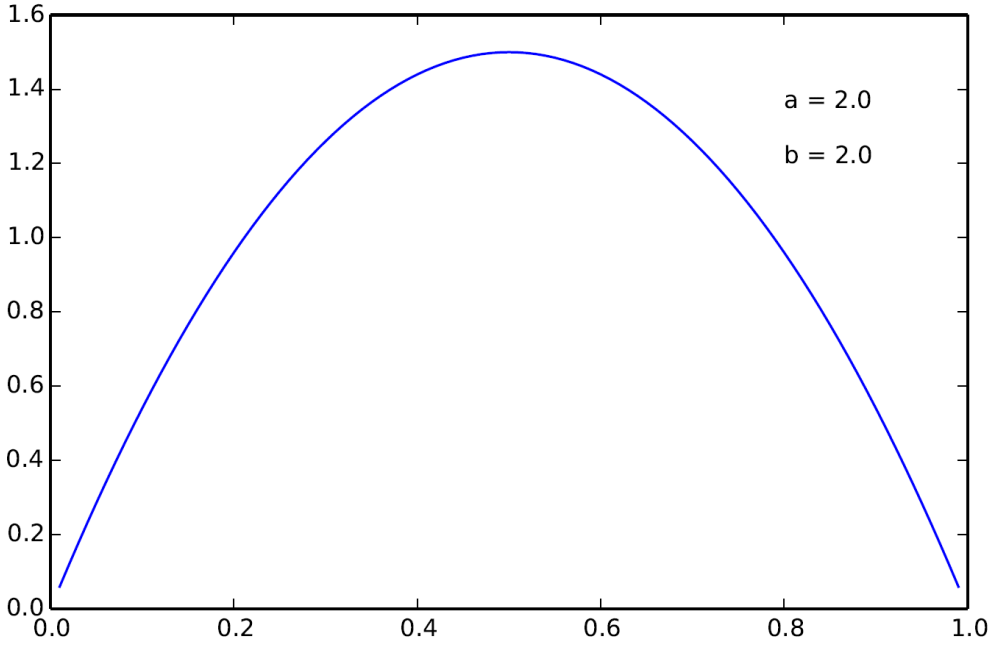
$$1 \geq \sum_{j=1}^{n} FF_{ij} \geq 0$$

Beta and Dirichlet distributions

- Lognormal
  - 3 degrees of freedom: minimum (often zero), average, variance
  - Only has a lower bound
  - skewed to higher values
  - VERY skewed for high variance
- Beta
  - Upper and lower bound
  - 4 degrees of freedom: min, max, α, β
  - Choose min, max, average, percentile 97.5 → α, β
  - Skewed to the right, left, or symmetrical

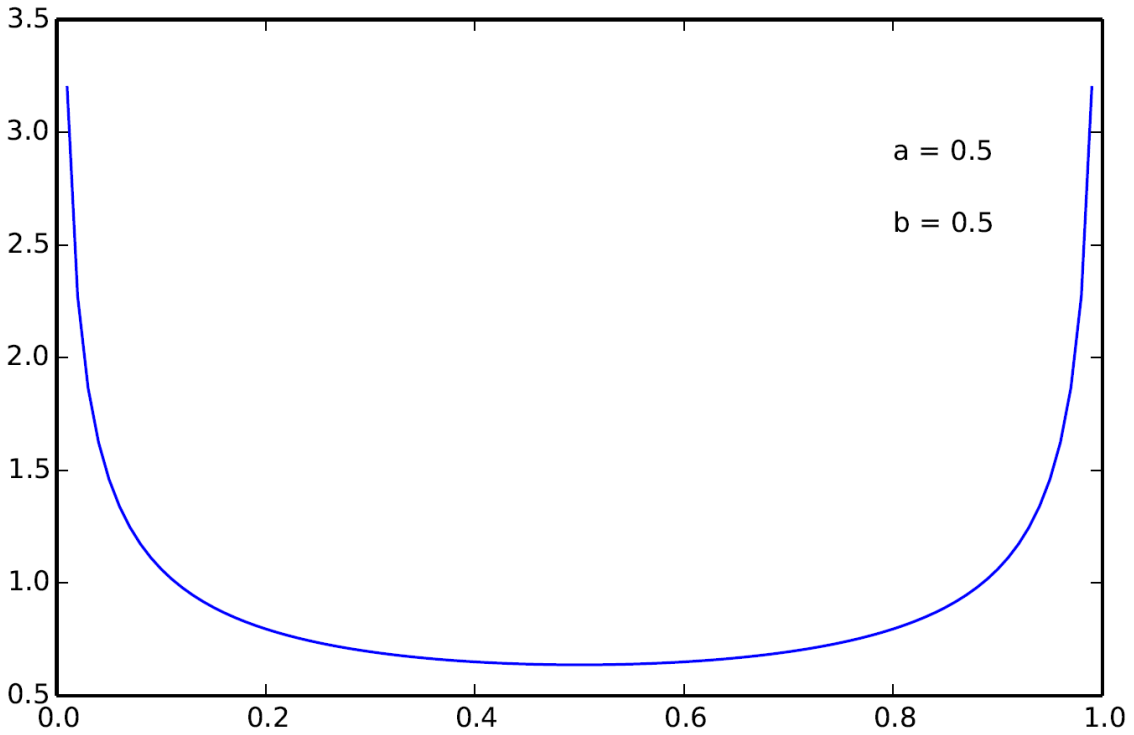
Beta and Dirichlet distributions



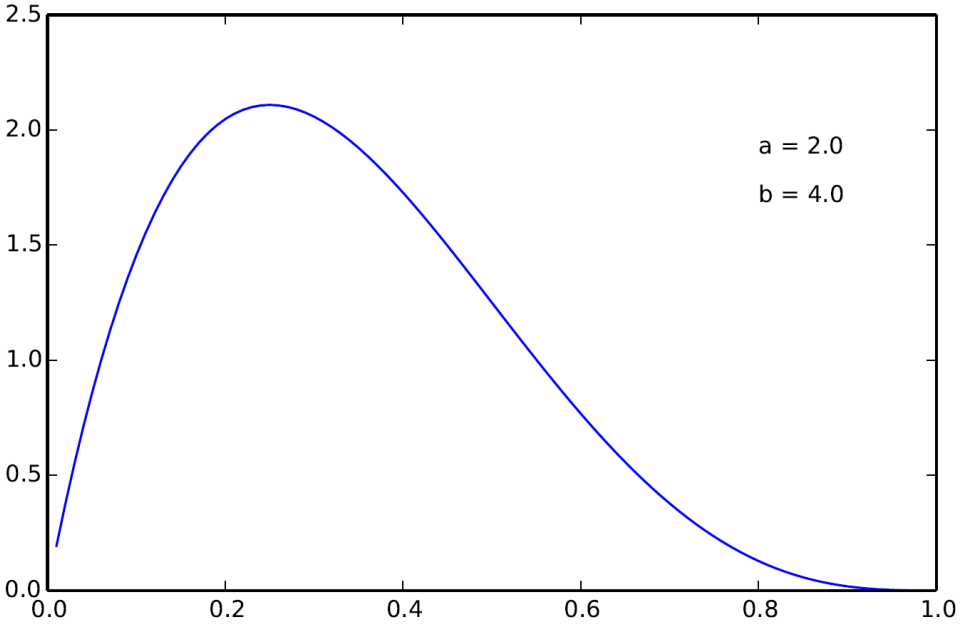


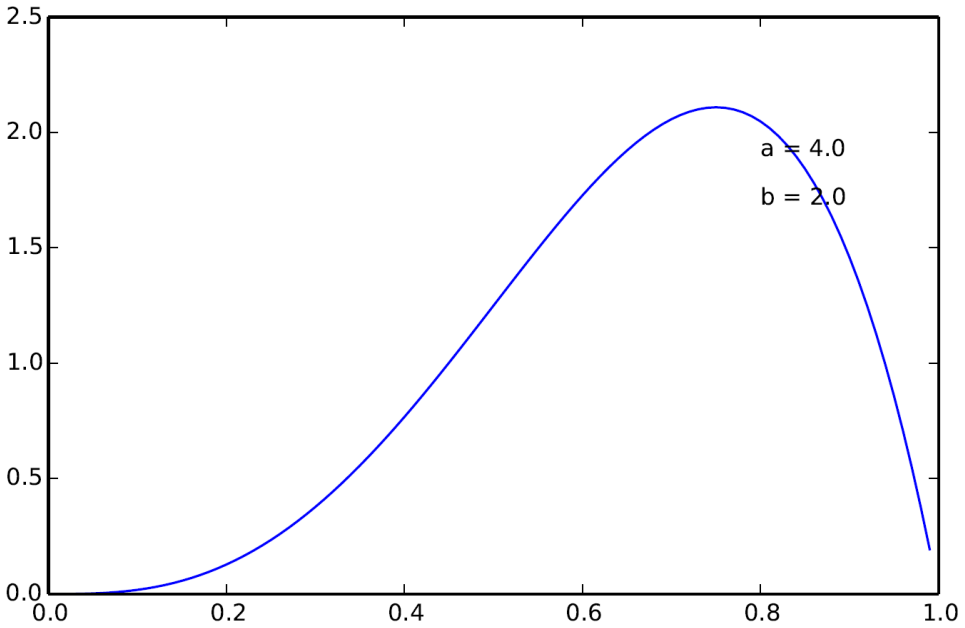
$$\alpha + \beta \downarrow, \sigma \uparrow$$

Beta and Dirichlet distributions



Beta and Dirichlet distributions





Beta and Dirichlet distributions

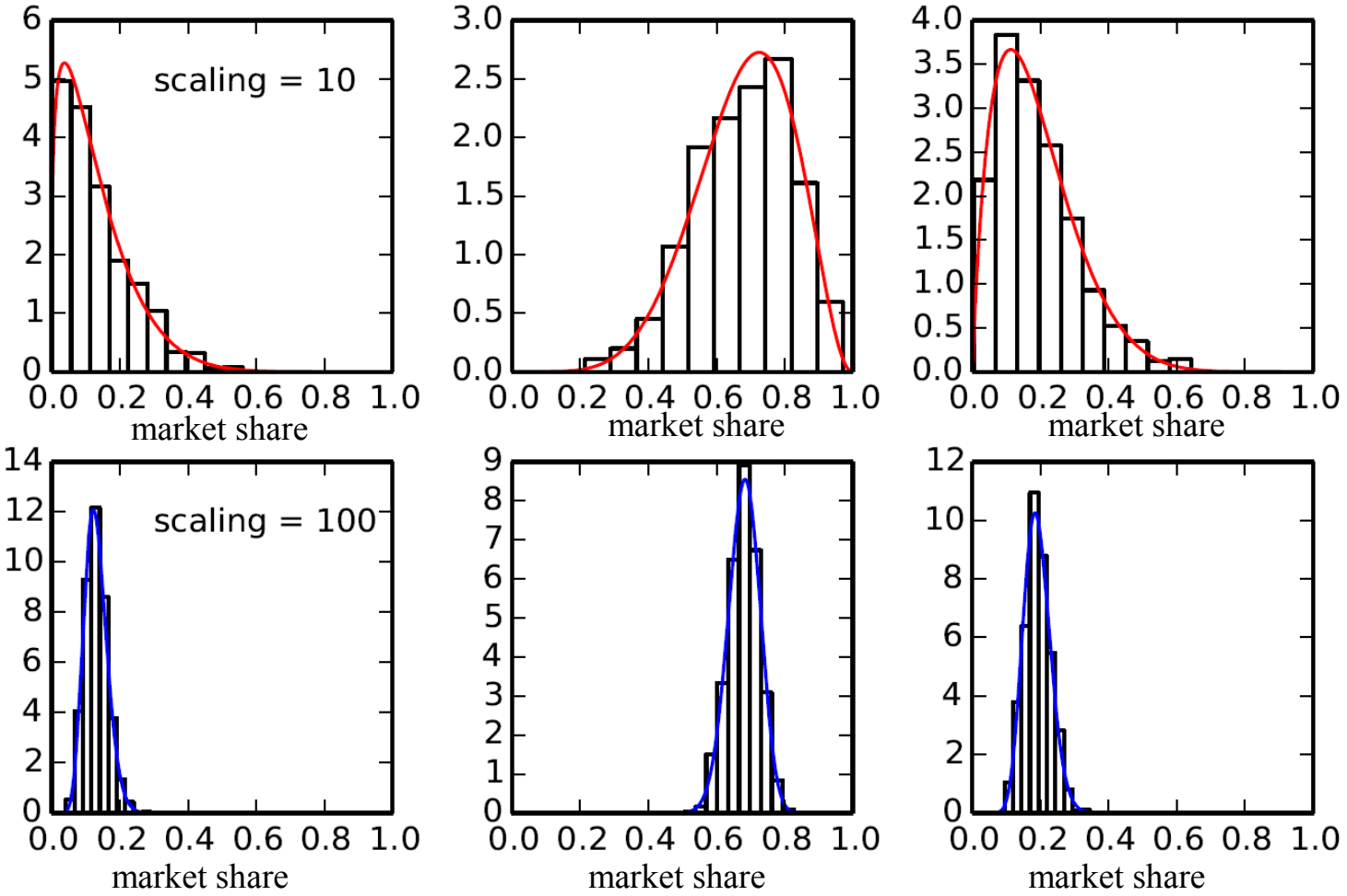
- Dirichlet is a multi-variate beta
- Inputs of Dirichlet:
  - Vector 1 x N. Example: [0.13, 0.68, 0.19]
  - Scaling value. Example: 10 and 100
  - M iteration. Example: 1000
- Output of Dirichlet:
  - Parameters for N beta distributions
  - N samples of M iterations

Beta and Dirichlet distributions

- Each iteration sums to 1

```
Out[65]:
array([[ 0.22502527,  0.59061152,  0.18436321],
       [ 0.00830886,  0.7870208 ,  0.20467033],
       [ 0.1394482 ,  0.58871838,  0.27183342],
       ...,
       [ 0.06549313,  0.85525442,  0.07925245],
       [ 0.10132079,  0.87050497,  0.02817424],
       [ 0.02473596,  0.45133565,  0.52392839]])
```

Beta and Dirichlet distributions

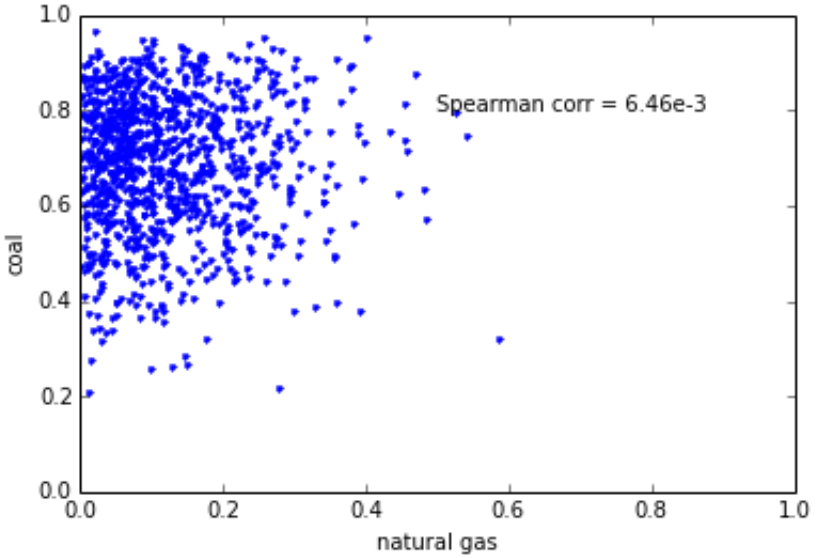


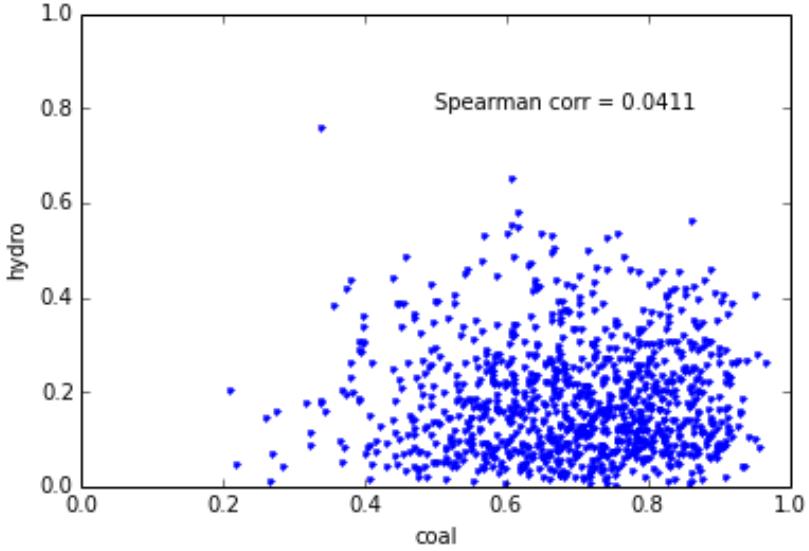
Beta and Dirichlet distributions

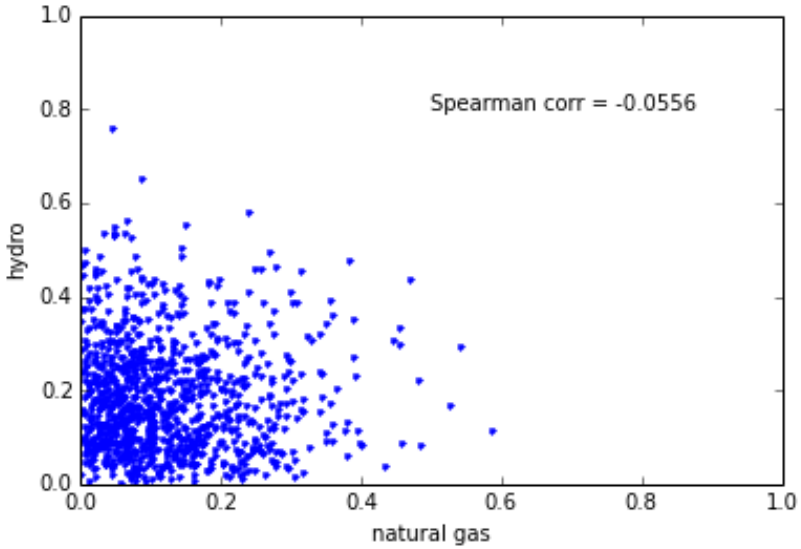
- Shuffled sample

```
Out[72]:
array([[ 0.11361161,  0.61693219,  0.28884694],
       [ 0.11066555,  0.87386014,  0.40323989],
       [ 0.08209619,  0.62743918,  0.10454758],
       ...,
       [ 0.01507658,  0.57664339,  0.33551239],
       [ 0.07420796,  0.61165905,  0.23939245],
       [ 0.13773725,  0.74982074,  0.10817512]])
```

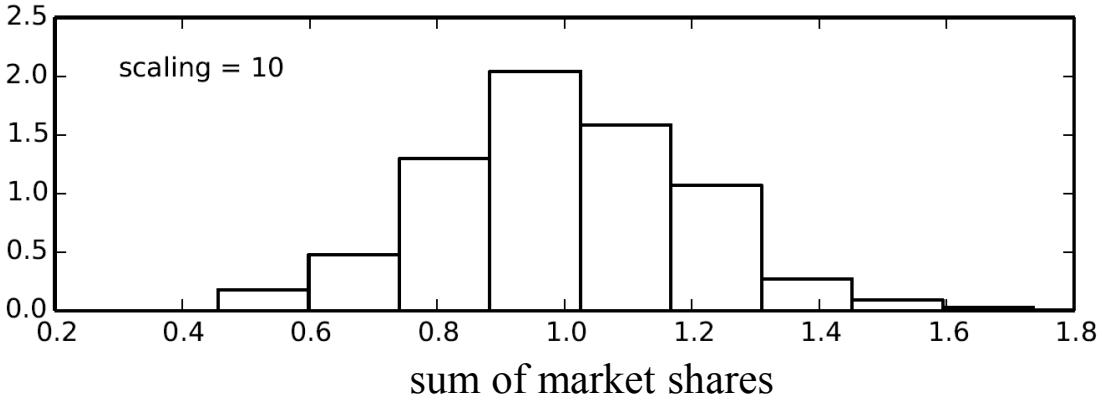
Beta and Dirichlet distributions

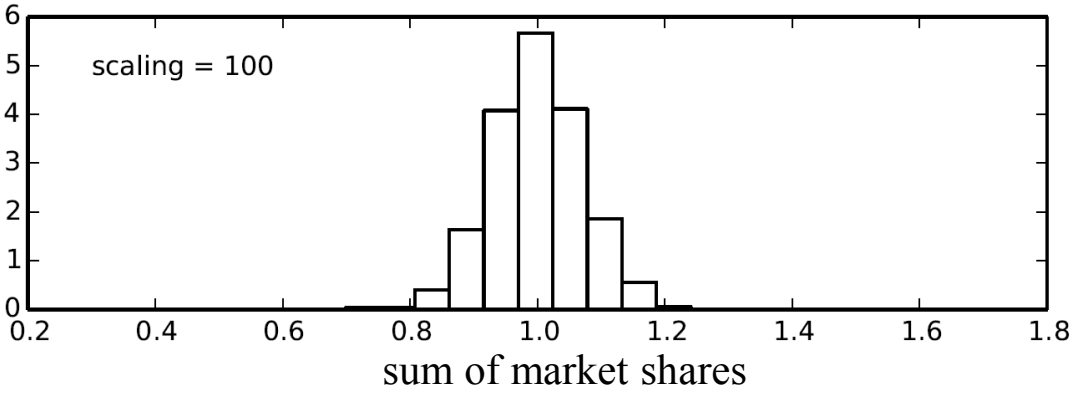






Beta and Dirichlet distributions

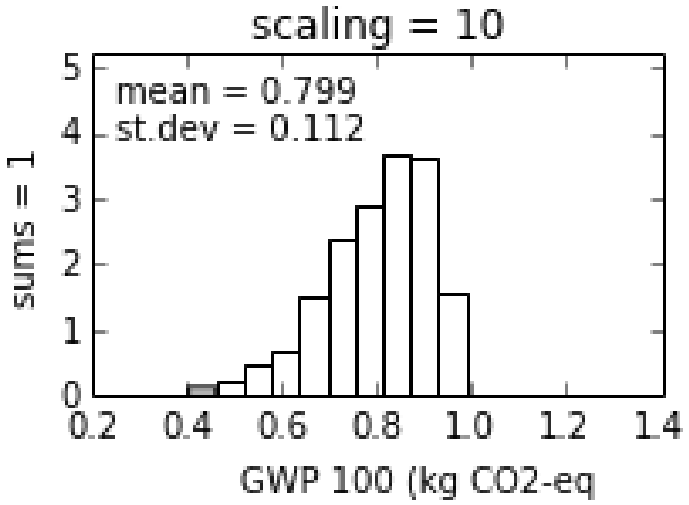


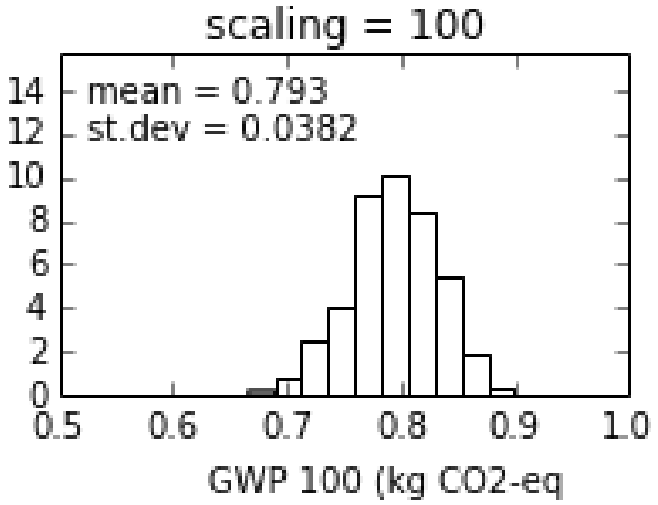


Beta and Dirichlet distributions

- Electricity production, ecoinvent v3.1, IPCC 2007 GWP 100 scores, Austria
  - Natural gas : 0.84362 kg CO_2-eq/kWh
  - Hard coal : 1.0022 kg CO_2-eq/kWh
  - Hydro, run-of-river: 0.0044188 kg CO_2-eq/kWh

Beta and Dirichlet distributions





Take home message

- Ignoring correlation leads to uncertainty overestimation
- Larger the uncertainty of the parameters → larger overestimation
- No rule of thumb to predict if the difference is noticeable
- Safer to test if the effect is significant in the setting at hand
- Ask your software provider the detail of the algorithm
- Ask a statistician friend!

The world's most consistent and transparent Life Cycle Inventory database

ecoinvent

Thank you for your attention!
Questions?

Guillaume Bourgault

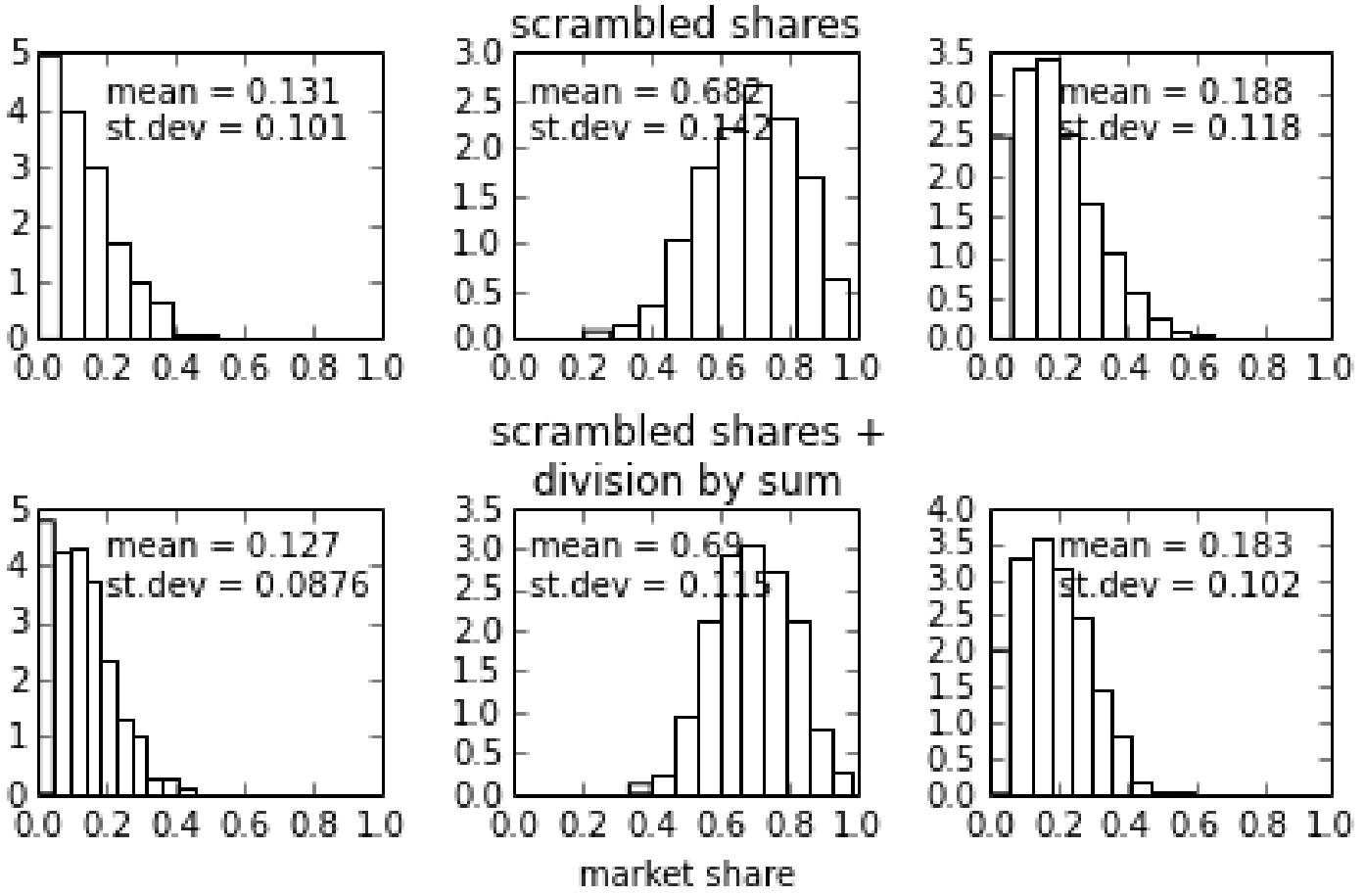
Project Manager

bourgault@ecoinvent.org

www.ecoinvent.org

Agroscope EMPA EPFL ETH PSI

Why not divide by sum?



Why not divide by sum?